Math 10A
Quiz 3; Friday, 6/29/2018
Time: 3 PM
Instructor: Roy Zhao **Name:** ____________________

Circle True or False. (1 point for correct answer, 0 if incorrect)

1. True **FALSE** The second derivative test will always tell us whether a critical point is a local minimum, local maximum, or neither.

> **Solution:** If the second derivative is zero, then the test fails.

2. **TRUE** False When we graph a function, the first derivative tells us if the function is increasing or decreasing and the second derivative tells us the concavity.

Show your work and justify your answers. Please circle or box your final answer.

3. (10 points) (a) (3 points) Find y' if $(x-y)^2 = x+y-1$.

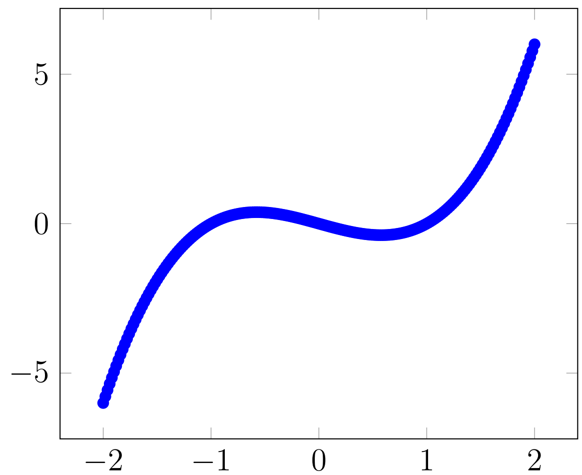
> **Solution:** Taking the derivative gives $2(x-y)(1-y') = 1+y'$ so $2x-2y-1 = y'(1+2x-2y)$ so $y' = \dfrac{2x-2y-1}{2x-2y+1}$.

(b) (7 points) Graph $f(x) = x^3 - x$ by finding the intervals of increasing/decreasing, concavity, etc. (Hint: $f(\sqrt{1/3}) \approx -0.4, f(-\sqrt{1/3}) \approx 0.4$)

Solution: Take the derivative and second derivative to get $f'(x) = 3x^2 - 1$ and $f''(x) = 6x$. We want to make a table and the values we care about are when $f'(x) = 0$, $f''(x) = 0$, and when they are not defined. They are always defined and solving $f'(x) = 0$ gives $x^2 - 1/3 = 0$ so $x = \pm 1/\sqrt{3}$, and $f''(x) = 0$ gives $x = 0$. So the points we need to put in our table are $x = 0, \pm 1/\sqrt{3}$. We fill out the table the sign of f', f'' on these intervals to get

	$(-\infty, -1/\sqrt{3})$	$-1/\sqrt{3}$	$(-1/\sqrt{3}, 0)$	0	$(0, 1/\sqrt{3})$	$1/\sqrt{3}$	$(1/\sqrt{3}, \infty)$
$f'(x)$	$-$	0	$+$	$+$	$+$	0	$-$
$f''(x)$	$+$	$+$	$+$	0	$-$	$-$	$-$

Now we calculate the limits as $x \to \pm\infty$. We have $\lim_{x\to-\infty} f(x) = -\infty$, $\lim_{x\to\infty} f(x) = \infty$. We can now use this to produce something similar to the following graph noting that f will have a local minimum at $x = -1/\sqrt{3}$ and maximum at $x = 1/\sqrt{3}$ by the second derivative test.


5
0
−5
−2
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2